I-3. LIGHT BEAM PROPAGATION IN CURVED SCHLIEREN GUIDES

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Schlieren guides consisting of a suitable transverse profile of optical density in a gas may not only be used to continuously focus light beams of Hermite-Gaussian field distribution but also to guide such beams around bends.

From the ray equation of geometric optics in the toroidal coordinates of a bent cylindrical schlieren structure (Figure 1) equations of motion are found for the transverse beam center vector $\vec{R}$ and the transverse beam radius vector $\vec{a}$. In the euqation for $\vec{a}$ diffraction spreading is subsequently taken into account.

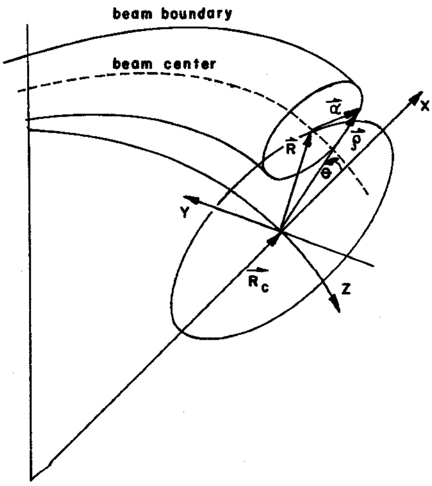

Figure 1. Light Beam Propagation in Toroidal Coordinates (ρ,θ,z)

For a uniform schlieren structure with

$$n = n(\rho) = n_o\left(1 - \frac{c\rho^2}{2}\right), \tag{1}$$

the beam displacement in the plane of the bend follows from

$$\frac{d^2x}{dz^2} = -cx + \frac{1}{R_c}, \tag{2}$$

where R_c is the radius of curvature. The equation of motion for $\vec{a}$ is independent of $\vec{R}$ and $\vec{R}_c$. The solution of Equation (2) for a constant curvature and coaxial and paraxial beam at the input is

$$x = \frac{1}{c R_c} \left[1 - \cos \sqrt{c}\ z\right] \text{ (Figure 2).} \qquad (3)$$

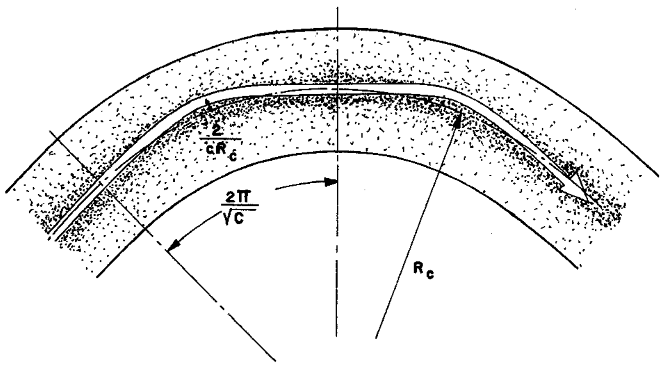

Figure 2. Beam Displacement in a Uniform Schlieren Guide of Specific Convergence c and Constant Curvature $1/R_c$

Displacing the input beam by $x = \frac{1}{c R_c}$ eliminates undulations of the beam (Figure 3). Tapered curvature bends (Figure 4) reduce undulations when $\sqrt{c}L > 1$. Large beam deflections are caused by sinusoidal curvature distributions of period $\frac{2\pi}{\sqrt{c}}$ or any distributions with Fourier components at this period-length. For random curvature distributions, the r.m.s. of beam deflection is

$$x^{-2} = \sqrt{\frac{z}{2c}}\ \phi(\sqrt{c}) \qquad (4)$$

where $\phi(\sqrt{c})$ is the power spectrum of $\frac{1}{R_c(z)}$ at the spectral component of the spatial frequency $\sqrt{c}$. Experimental models of uniform schlieren guide (Reference 1) with 8 mm I.D. have shown a specific convergence $c = 0.2\ m^{-2}$. For the beam not to hit the wall in this guide: in case of constant curvature $R_c \geq 2500$ m, in case of tapered curvature $R_c \geq 1250$ m and in case of random curvature of exponential covariance and the most critical correlation distance $\frac{1}{\sqrt{c}} = 2.2$ m: $\sqrt{\overline{R^{-2}}_c} \geq 60$ km.

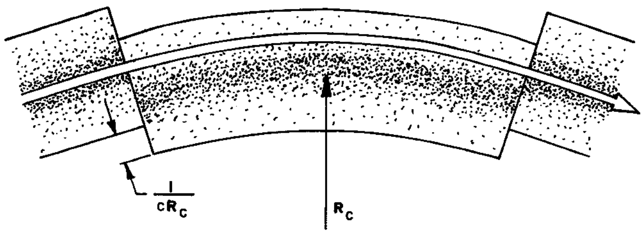

Figure 3. Constant Beam Displacement for Matched Input and Output Conditions in a Constant Curvature Bend

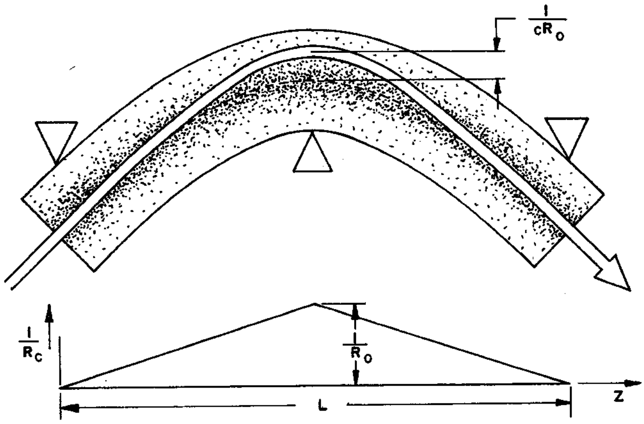

Figure 4. Linear Curvature Tapers of a Triangular Curvature Distribution Reduce Beam Undulations

A periodic schlieren structure with

$$n = n_o \left[1 - (c_0 + c_1 \cos 2\pi \frac{z}{p}) \frac{\rho^2}{2}\right] \tag{5}$$

is a suitable model for a schlieren guide with tubular gas lenses (Figure 5). Here the beam displacement follows from

$$\frac{d^2x}{dz^2} = (c_0 + c_1 \cos 2\pi \frac{z}{p})\, x + \frac{1}{R_c} \tag{6}$$

while y and $\vec{a}$ are independent of x and R_c. Equation (6) is an inhomogeneous Mathieu equation. For $\frac{c_1 p^2}{2\pi^2} \ll 1$ and $\frac{c_0 p^2}{\pi^2} \ll 1$ the periodic schlieren structure is with respect to beam displacement in curvature and beam focusing equivalent to a uniform schlieren guide with specific convergence

$$c = c_0 + \frac{1}{2}\left(\frac{c_1 p}{2\pi}\right)^2 \tag{7}$$

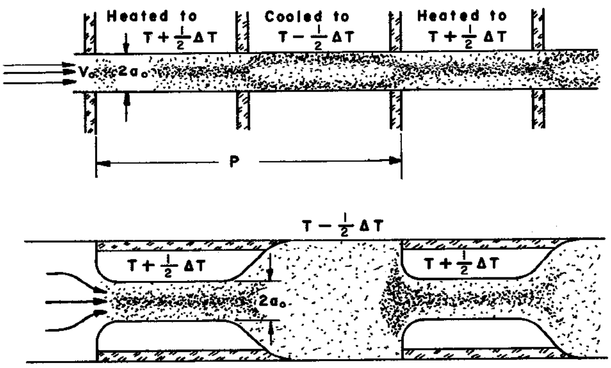

Figure 5. Periodic Schlieren Guides of Alternatingly Heated and Cooled Pipe Sections

One tubular gas lens in Figure 5 is with respect to focusing and beam guiding characteristics nearly equivalent to a uniform schlieren structure which under optimum conditions has a specific convergence (Reference 2),

$$c' = 1.68 \frac{\Delta T}{T} \frac{n_o - 1}{a_o^2} . \tag{8}$$

For alternatingly heated and cooled sections of equal diameter: $c_0 = 0$ and $c_1 = c'$ in Equation (5), for much wider cooled sections $c_0 = c_1 = \frac{c'}{2}$. The first structure has alternating gradient, in the second the gradient varies between zero and a maximum. For $\frac{\Delta T}{T} = 0.1$, $a_o = 4$ mm, CO_2 streaming with $v_o = 2 \frac{m}{sec}$ and optimum length p, the equivalent specific convergence of the first structure is $c = 0.182\ m^{-2}$ while of the second structure it is $c = 2.2\ [1 + 0.0211]\ m^{-2}$. The second term in the brackets is quite small indicating that the alternating gradient focusing contributes only little to the equivalent specific convergence in this case. With $c = 2.2\ m^{-2}$ in a 10 km long schlieren guide of $a_o = 4$ mm, the r.m.s. radius of random curvature of exponential covariance and most critical correlation distance $\frac{1}{\sqrt{c}} = 0.67$ m must be more than 10 km for the r.m.s. beam deflection not to hit the wall.

A quadruple-wire helix (Figure 6) with pairs of helix wires at temperature differences ΔT will by thermal diffusion create a helical schlieren structure of refractive index (Reference 3),

$$n = n_o \left[1 - \frac{c_h}{2} \rho^2 \cos\left(\frac{2\pi z}{p} - \theta\right)\right] , \tag{9}$$

where $c_h = (n_o - 1) \frac{\Delta T}{2T\ a_o^2}$. The rectangular components of the equations of motion for the beam center vector are in this case:

$$\frac{d^2x}{dz^2} = -c_h \left[x \cos \frac{2\pi z}{p} + y \sin \frac{2\pi z}{p}\right] + \frac{1}{R_c} \tag{10}$$

$$\frac{d^2y}{dz^2} = -c_h \left[x \sin \frac{2\pi z}{p} - y \cos \frac{2\pi z}{p}\right] . \tag{11}$$

The beam radius equation is again independent of beam displacement and guide curvature. For input beams in the plane of the bend always $y(z) \ll x(z)$ if only $\frac{c_h}{2} \frac{p^2}{\pi^2} \ll 1$. Equations (10) and (11) can under this condition be solved by iteration. Neglecting at first the y-term in Equation (10) this is a degenerate inhomogeneous Mathieu equation. Its solution for any initial conditions of x substituted into Equation (11) makes this equation also an inhomogeneous Mathieu equation.

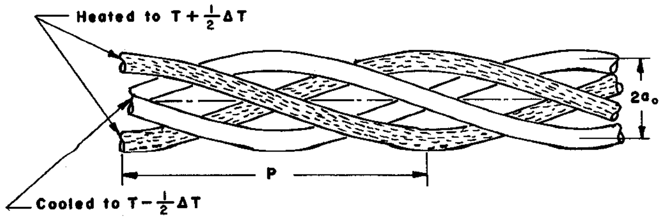

Figure 6. Helical Schlieren Guide from Thermal Diffusion in a Quadruple-Wire Helix of Different Temperatures

With respect to beam displacement in curvature and also with respect to beam mode propagation, the helical schlieren guide is equivalent to a uniform schlieren guide of specific convergence

$$c = \frac{1}{2}\left(\frac{c_h p}{2\pi}\right)^2 \tag{12}$$

For a quadruple-wire helix with a_o = 4 mm, p = 1 m at $\frac{\Delta T}{T}$ = 0.1 surrounded by CO_2 at 3 atmospheres of pressure the equivalent specific convergence is $c = 0.2\ m^{-2}$.

Hence, with all schlieren guides which have so far been studied theoretically or experimentally uniform curvature of typically between 300 and 3000 m radius of curvature may be tolerated while for random curvature the spectral components of $\frac{2\pi}{\sqrt{c}}$ period length have to be kept extremely small. Furthermore, in all these guides Hermite-Gaussian beam modes propagate as in a straight uniform schlieren guide of the equivalent specific convergence no matter what the curvature is and how much it displaces the beam center off the axis.

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